A CGM APPROACH TO SUBSPACE BASED BLIND CHANNEL IDENTIFICATION

*Nari TANABE**, *Toshihiro FURUKAWA*†, *Shigeo TSUJII*‡,

*Toyama National College of Technology, 13 Hongo-machi, Toyama 939–8630, Japan.
†Science University of Tokyo, 1–3 Kagurazaka, Shinjuku-ku, Tokyo 162–0825, Japan.
‡Institute of information Security, 2–14–1 Tsuruya-cho, Kanagawa-ku, Yokohama 221–0835, Japan.

ABSTRACT

We propose a CGM (conjugate gradient method) approach to subspace based blind channel identification. The algorithm estimates (1) the channel order, (2) the noise variance, (3) the noise subspace, and them identifies (4) channel impulse response without using the eigenvalue decomposition. The special features of the proposed algorithm are (1) accurate channel order estimation and (2) the reduction of computational complexity using CGM. Numerical examples show the effectiveness of the proposed algorithm.

1. INTRODUCTION

A blind channel identification algorithm is a major concern to most researchers in various signal processing and communication fields [1, 2, 3, 4, 5]. Among then, Moulines' method based on the second order statistics of the received signal is most effective approaches to blind channel identification. The Moulines' method [2] gives good performance and well works under the noisy environment by the use of the principal component analysis. However, the Moulines' method requires (1) accurate channel order estimation in fact that its performance is extremely sensitive to channel order mismatch, and (2) a rather large amount of computation for eigenvalue decomposition and the high computational complexity become disadvantageous for its adaptive implementation.

For those problems, Abed-Meraim et al. [3] proposed linear prediction algorithm with handled an over-estimated channel order, while its performance is very sensitive to the observation noise. Moreover, Liavas et al. [4] have proposed channel order estimation based on eigenvalue decomposition. However, its approach still needs high *SNR* for good performance. Although Yang [5] proposed the subspace tracking method without using eigenvalue decomposition for signal and noise subspaces estimation, its approach has made an unrealistic assumption that channel order is known.

To overcome the drawbacks, we propose a new algorithm for estimating the channel impulse response using second order statistics of received signal. The algorithm estimates the channel order, the noise variance, the noise subspace, and then channel impulse response using CGM (conjugate gradient method). By avoiding the use of eigenvalue decomposition, the proposed algorithm reduces the computational complexity. It is shown through numerical examples that the proposed algorithm is quite effective and practical.

2. PROBLEM FORMULATION

Let $x(t)$ be the received signal at noisy communication channel with impulse response $h(t)$, i.e.,

$$x(t) := d(t) * h(t) + v(t) \tag{1}$$

where $d(t) := \sum_n d_n \delta(t - nT)$ denotes the information signal to be transmitted every T, $*$ the convolution and $v(t)$ the additive observation noise given by $AWGN$. The transmitted information symbol d_ℓ is assumed to satisfy

$$E\left[d_\ell\right] = 0,\ E\left[d_\ell d_m\right] = \delta_{\ell m},\ E\left[d_\ell v(t)\right] = 0. \tag{2}$$

For the blind channel identification using second order statistics of received signal, the channel output signal $x(t)$ is oversampled at $t = nT + \frac{P-1}{P}T$ to get

$$x_n^{(p)} = \sum_{\ell=0}^{L} h_\ell^{(p)} d_{n-\ell} + v_n^{(p)} \tag{3}$$

where $x_n^{(p)} := x(nT + (p-1)T/P)$, $h_n^{(p)} := h(nT + (p-1)T/P)$ and $v_n^{(p)} := v(nT + (p-1)T/P)$ for $p = 1, 2, \cdots, P$.

It is convenient to express Eq.(3) in terms of vector form:

$$\boldsymbol{x}_n = \sum_{\ell=0}^{L} \boldsymbol{h}_{n-\ell} d_n + \boldsymbol{v}_n \tag{4}$$

where $\boldsymbol{x}_n := [x_n^{(1)}, x_n^{(2)}, \cdots, x_n^{(P)}]$, $\boldsymbol{h}_n := [h_n^{(1)}, h_n^{(2)}, \cdots, h_n^{(P)}]$ and $\boldsymbol{v}_n := [v_n^{(1)}, v_n^{(2)}, \cdots, v_n^{(P)}]$.

Furthermore, let a given positive integer N, we define a received signal vector $\boldsymbol{x} := [\boldsymbol{x}_n^T,\ \boldsymbol{x}_{n-1}^T,\ \cdots,\ \boldsymbol{x}_{n-N}^T]^T$ from Eq.(4) to get matrix form:

$$\boldsymbol{x} := \mathcal{H}_N \boldsymbol{d} + \boldsymbol{v} \tag{5}$$

where $\boldsymbol{d} := [d_n,\ d_{n-1},\ \cdots,\ d_{n-L-N}]^T$, $\boldsymbol{v} := [\boldsymbol{v}_n^T,\ \boldsymbol{v}_{n-1}^T,\ \cdots,\ \boldsymbol{v}_{n-N}^T]^T$, and $\mathcal{H}_N$ is an $P(N+1)\times(L+N+1)$ block Toeplitz matrix called as the *Sylvester matrix* [1, 2] of order N associated with the $P(L+1)\times 1$ vector $\boldsymbol{h} := [\boldsymbol{h}_0^T,\ \boldsymbol{h}_1^T,\ \cdots,\ \boldsymbol{h}_L^T]^T$.

The blind channel identification problem consists in estimating the $P(L+1)\times 1$ vector $\boldsymbol{h}$ form the second order statistics of received signal $\boldsymbol{x}$, which can be written under the assumption of Eq.(2) as

$$R_x := E\left[\boldsymbol{x}\boldsymbol{x}^T\right] = \mathcal{H}_N\mathcal{H}_N^T + \sigma_v^2 I_\eta \tag{6}$$

where $E[\cdot]$ denotes the ensemble average and σ_v^2 is noise variance of $v(t)$. The *SNR* (signal to noise radio) is defined by

$$10^{\frac{SNR}{10}} = \frac{Tr[R_x]}{P(N+1)\sigma_v^2} - 1 = \frac{1}{P}\sum_{p=1}^{P}\frac{\|\boldsymbol{h}^{(p)}\|^2}{\sigma_v^2} \tag{7}$$

In this paper, we exclusively use the symbols, $\eta := P(N+1)$ and $\rho := L+N+1$, for simplicity.

3. CONVENTIONAL METHOD [2]

Under the condition that $\mathcal{H}_N$ is full column rank, that is rank $(\mathcal{H}_N) = \rho$, Eq.(6) can be diagonalization as

$$\begin{aligned} R_x &= U_x\Lambda_x U_x^T \\ &= [U_s U_v]\begin{bmatrix}\Lambda_s + \sigma_v^2 I_\rho & O \\ O & \sigma_v^2 I_{\eta-\rho}\end{bmatrix}\begin{bmatrix}U_s^T \\ U_v^T\end{bmatrix} \\ &= U_s \text{diag}\ (\lambda_1^{(s)},\ \lambda_2^{(s)},\ \cdots,\ \lambda_\rho)U_s^T + \sigma_v^2 U_v U_v^T, \end{aligned} \tag{8}$$

where

$$\left.\begin{aligned} &U_x := [U_s,\ U_v],\ \Lambda_x := \text{diag}\ (\lambda_1,\ \lambda_2,\ \cdots,\ \lambda_\eta), \\ &U_s := [\boldsymbol{u}_1,\ \boldsymbol{u}_2,\ \cdots,\ \boldsymbol{u}_\rho],\ U_v := [\boldsymbol{u}_{\rho+1},\ \boldsymbol{u}_{\rho+2},\ \cdots,\ \boldsymbol{u}_\eta], \\ &\Lambda_s := \text{diag}\ (\lambda_1^{(s)},\ \lambda_2^{(s)},\ \cdots,\ \lambda_\rho^{(s)}),\ \lambda_i^{(s)} > 0, \end{aligned}\right\} \tag{9}$$

and the column-space of $\mathcal{H}_N$, the so-called *signal subspace*, is the orthogonal complement of the column-space of U_v, the so-called *noise subspace*.

It is easily seen that

$$\boldsymbol{u}_i^T\mathcal{H}_N = \boldsymbol{0}, \quad \text{for } i = \rho+1,\ \rho+2,\ \cdots,\ \eta. \tag{10}$$

Therefore, by using the structural identity $\boldsymbol{u}_i^T\mathcal{H}_N = \boldsymbol{h}^T\mathcal{U}_L^{(i)}$[1, 2], Eq.(10) can be rewritten as

$$\boldsymbol{h}^T\left[\mathcal{U}_L^{(\rho+1)},\ \mathcal{U}_L^{(\rho+2)},\ \cdots,\ \mathcal{U}_L^{(\eta)}\right] = \boldsymbol{0}^T \tag{11}$$

Table 1. CGM procedure

Initial value: $\ell = 1$
1. $\boldsymbol{z}_1(s) = \boldsymbol{0}$
2. $\boldsymbol{q}_1(s) = \boldsymbol{b}(s) - A(s)\boldsymbol{z}_1(s)$
3. $\boldsymbol{g}_1(s) = \boldsymbol{q}_1(s)$
Iteration: for $\ell = 2,\ 3,\ \cdots,\ \eta$
4. $\alpha_\ell(s) = \frac{\boldsymbol{q}_\ell(s)^T\boldsymbol{g}_\ell(s)}{\boldsymbol{g}_\ell(s)^T A(s)\boldsymbol{g}_\ell(s)}$
5. $\boldsymbol{z}_{\ell+1}(s) = \boldsymbol{z}_\ell(s) + \alpha_\ell(s)\boldsymbol{g}_\ell(s)$
6. $\boldsymbol{q}_{\ell+1}(s) = \mathbf{q}_\ell(s) - \alpha_\ell(s)A(s)\boldsymbol{g}_\ell(s)$
7. if $\|\boldsymbol{q}_{\ell+1}(s)\| < \varepsilon$ then break, else go to 8
8. $\beta_\ell(s) = \frac{\boldsymbol{q}_{\ell+1}(s)^T\boldsymbol{q}_{\ell+1}(s)}{\boldsymbol{q}_\ell(s)^T\boldsymbol{q}_\ell(s)}$
9. $\boldsymbol{g}_{\ell+1}(s) = \boldsymbol{q}_{\ell+1}(s) + \beta_\ell\boldsymbol{g}_\ell(s)$

where $\mathcal{U}_L^{(i)}$ is the $P(L+1)\times\rho$ *Sylvester matrix* of order L associated with $\boldsymbol{u}_i^T = [\boldsymbol{u}_0^{(i)^T},\ \boldsymbol{u}_1^{(i)^T},\ \cdots,\ \boldsymbol{u}_N^{(i)^T}]$.

The solution is conventionally calculated as the vector $\boldsymbol{h}$ that minimizes

$$J(\boldsymbol{h}) := \sum_{i=\rho+1}^{\eta}\|\boldsymbol{h}^T\mathcal{U}_L^{(i)}\|^2 = \boldsymbol{h}^T\left(\sum_{i=\rho+1}^{\eta}\mathcal{U}_L^{(i)}\mathcal{U}_L^{(i)^T}\right)\boldsymbol{h} \tag{12}$$

subject to $\|\boldsymbol{h}\| = 1$.

However, Moulines' method [1, 2] requires a rather large amount of computation for eigenvalue decomposition in Eqs.(8) and (12). Furthermore, it is difficult to determine the value of ρ (in Eq.(8) or Eq.(9)) to obtain the channel order $L(= \rho - N - 1)$ in a practical system. Recently, Liavas [4] proposed a criterion for discriminating λ_ρ and $\lambda_{\rho+1}$, while it still requires high *SNR* for the correct channel order estimation.

4. PROPOSED METHOD

For the autocorrelation matrix R_x in Eq.(6) and a given s (an estimate of noise variance σ_v^2), let

$$A(s) := R_x - sI_\eta = \mathcal{H}_N\mathcal{H}_N^T + (\sigma_v^2 - s)I_\eta. \tag{13}$$

Then perform the CGM procedure [8] in Table 1 for

$$A(s)\boldsymbol{z}(s) := \boldsymbol{b}(s) \tag{14}$$

where $\boldsymbol{z}(s)$ is the $\eta\times 1$ unknown vector and $\boldsymbol{b}(s)$ the $\eta\times 1$ vector given by sum of all column vectors in $A(s)$, i.e.,

$$\boldsymbol{b}(s) := A(s)[1,\ 2,\ \cdots,\ \eta]^T. \tag{15}$$

Note that our goal is not the estimation of optimum solution $\boldsymbol{z}_{opt}(s)$ in Eq.(14) but channel order, noise variance, noise subspace and then channel impulse response by calculating Eq.(14) using the property of CGM.

In the next subsection, we briefly introduce the proposed method without using eigenvalue decomposition.

4.1. Estimation of Channel Order

When we define the function $f(s) := \|\boldsymbol{q}_\ell(s)\|^2$ using a residual vector $\{\boldsymbol{q}_\ell(s)\}_\ell$ in Table 1, perform the CGM for Eq.(14). Then the CGM can obtain the good solution $\boldsymbol{z}_{opt}(s)$ if the iteration number of CGM is more than the number of the different eigenvalue, denoted by γ, of the matrix A(s), that is, $f(s) = \|\boldsymbol{q}_\gamma(s)\|^2 \simeq 0$ and $\gamma = \rho + 1$ from CGM property and Eq.(8) [6, 7, 8]. Hence, we can get the channel order $L(= \rho - N + 1)$ using $\rho = \gamma - 1$.

4.2. Estimation of Noise Variance

The shape of the $f(s)$ is as shown in general in Fig.1. Hence, we gather from Fig.1 as

[**Property of** $f(s)$]

(1) $f(s) = 0 \Leftrightarrow s = \lambda_i, \quad i = 1,\ 2,\ \cdots,\ \rho.$

(2) In the neighborhood of $s = \sigma_v^2$, it is convex and has unique minimum $f(\sigma_v^2)$.

Thus, the noise variance σ_v^2 is obtained as the minimum for function $f(s)$. Though there are several ways to research the noise variance σ_v^2, we can be found by following two step [9]:

First step (Estimation of range): Let $s_0 = 0$ and an appropriate step size $\Delta(> 0)$, we find $f(s_{i-1}) > f(s_i) > f(s_{i+1})$ by iterating $s_i := s_{i-1} + \Delta$.

Second step (Estimation of accurate value): For $s_{i-1} < \sigma_v^2 < s_{i+1}$, we may get a more accurate value of σ_v^2 by using the bisection method.

4.3. Estimation of Noise Subspace

If the noise variance σ_v^2 is good estimated in Section 4.2, the residual vectors $\boldsymbol{q}_\ell(s)$ in Table.1 has the property [6, 7, 8]:

[**Property of** $\boldsymbol{q}_\ell(s)$]

(1) $\boldsymbol{q}_\ell(s)^T \boldsymbol{q}_{k+1}(s) = 0, \quad \ell \le k.$

(2) When perform the CGM procedure in Table 1 for Eq.(14), let $Q := [\boldsymbol{q}_1(s), \boldsymbol{q}_2(s), \cdots, \boldsymbol{q}_\rho(s)]$. Then the column space of Q is expressed as

$$Span(Q) = Span(\mathcal{H}_N) = Span(A(\sigma_v^2))$$

under the condition of Eq.(15).

Thus, the basis of noise subspace, denoted by $\{\boldsymbol{v}_i\}_{i=1}^{\eta-\rho}$, can be found by

$$\boldsymbol{v}_i = (I - Q_\rho Q_\rho^T)\boldsymbol{y}_i \qquad (16)$$

from Eq.(10) and Property of $\boldsymbol{q}_\ell(s)$, i.e., $Span([\boldsymbol{v}_1,\ \boldsymbol{v}_2,\ \ldots,\ \boldsymbol{v}_{\eta-\rho}]) = Span(U_v)$, where $\boldsymbol{y}_i$ is random vector.

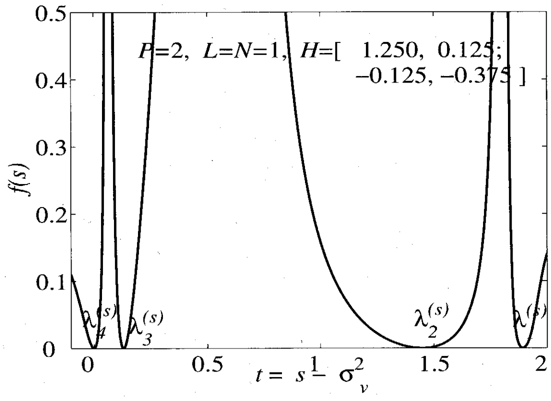

Fig. 1. Typical behavior of $f(s) = \|\boldsymbol{q}_\rho(s)\|^2$

4.4. Estimation of Impulse Response

The channel impulse response can estimate based on Eq.(11) and the previous subsection. That is, the vectors $\{\boldsymbol{v}_i\}$ (span noise subspace $Span(U_v)$) can construct the set of linear equations with unknown vector $\boldsymbol{h}$ in Eq.(11).

Though there are several ways to solve in Eq.(11), we shall employ in this paper as

(a) if $P = 2$ and $N = L$, $\boldsymbol{h}$ is simply and directly given by

$$\boldsymbol{h} := a\boldsymbol{u}_\ell^{(\eta)^T} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},\ \ell = 0, 1, \cdots, L$$

where $\boldsymbol{u}_\ell^{(\eta)^T}$ are the 2×1 component vectors for a basis of noise subspace in Eqs.(8) and (9), i.e., $\boldsymbol{u}_\eta^T =: [\boldsymbol{u}_0^{(\eta)^T}, \boldsymbol{u}_1^{(\eta)^T}, \cdots, \boldsymbol{u}_N^{(\eta)^T}]$,

(b) otherwise, directly solve Eq.(11) for $\boldsymbol{h}$ by Gaussian elimination subject to $\|\boldsymbol{h}\| = 1$,

to attain the minimum computational complexity.

5. NUMERICAL EXAMPLES

5.1. Simulation Conditions

The simulations are done for random real channel with white Gaussian information signal:

(i) The information signal $\{d_\ell\}$ to the communication channel is assumed to be zero mean, unit variance white Gaussian.

(i) The communication channel is given by random real channel with $h_\ell^{(p)}$ being independent zero mean and unit variance Gaussian variable.

We have considered here each case for P, L, and N, while for space we have shown only figures for a set of 100 randomly real channel with $P = 2$ and $L = 4$.

5.2. Simulation Result

To compare the performance of convergence speed and estimation accuracy for the algorithms, we show in Figs.2

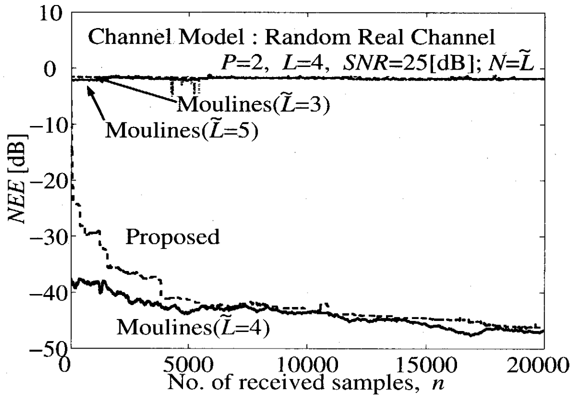

Fig. 2. Comparison of convergence behavior between the proposed and Moulines' methods.

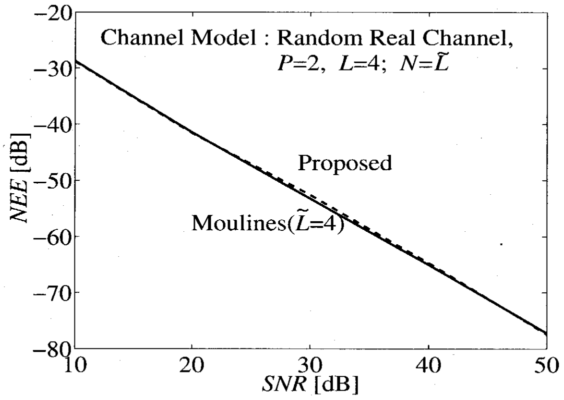

Fig. 3. Comparison of estimation accuracy behavior between proposed and Moulines' method.

and 3 the comparison of *NEE* (Normalized Estimation Error) for the channel impulse response defined by

$$NEE(n) := \frac{1}{100}\sum_{i=1}^{100} 10\log_{10}\left\| \frac{\boldsymbol{h}_i(n)}{\|\boldsymbol{h}_i(n)\|^2} - \frac{\widetilde{\boldsymbol{h}_i}(n)}{\|\widetilde{\boldsymbol{h}_i}(n)\|^2} \right\|^2 \quad [dB]$$

as performance measure. It is seen from Fig.2 that the proposed method can achieve exactly the same convergence speed as the Moulines' method with the correct channel order $L = 4$, its convergence speed can be improved by selecting the step size Δ.

We compare in Fig.3 estimation accuracy for the various *SNR* with 200 points average of iteration number (or number of received samples) $n = 19,801 \sim 20,000$. It is seen form Fig.3 that the proposed method is good performance and is similar to the Moulines' method.

We finally compare the complexity of proposed and Moulines' methods. We can see from Fig.4 that the proposed method required less computational complexity compared to Moulines' method.

Moreover, it is also easily guessed from Figs.2 and 3 that the proposed method can estimate the channel order correctly, and get the accurate noise variance σ_v^2 by increasing the iteration number.

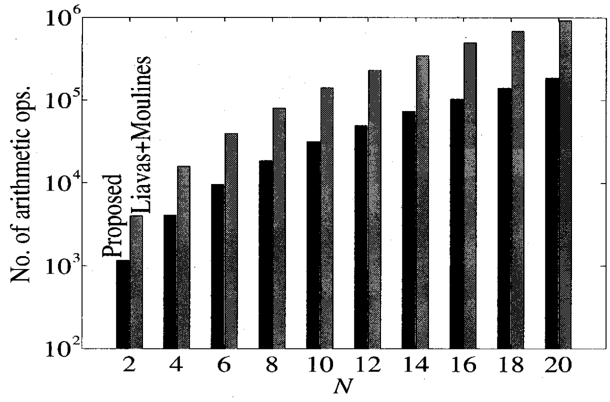

Fig. 4. Computational complexity in arithmetic operations.

6. CONCLUSION

We have shown that the proposed algorithm is effective using computer simulation. By avoiding the use of eigenvalue decomposition, the proposed method reduces the computational complexity without sacrificing the estimation accuracy and convergence speed.

References

[1] *Signal Processing Advances in Wireless and Mobile communications*, Volume 1: Trends in Channel Estimation and Equalization, Edited by G.B. Giannakis, et al. Prentice Hall, 2001.

[2] E. Moulines, P. Duhamel, "Subspace Methods for the Blind Identification of Multichannel FIR Filters," *IEEE Trans. on Signal Processing*, vol.43, no.2, pp.516–525, Feb. 1995.

[3] K. Abed-Meraim, E. Moulines and P. Loubaton, "Prediction Error Method for Second-Order Blind Identification," *IEEE Trans. on Signal Processing*, vol.45, no.3, pp.694–704, Mar. 1997.

[4] A. P. Liavas, P. A. Regalia and J-P. Delmas, "Blind Channel Approximation: Effective Channel Order Determination," *IEEE Trans. on Signal Processing*, vol.47, no.12, pp.3336–3344, Dec. 1999.

[5] B. Yang, "Projection Approximation Subspace Tracking," *IEEE Trans. on Signal Processing*, vol.43, no.1, pp.95–107, Jan. 1995.

[6] G. Strang, *Linear Algebra and Its Applications*, 3rd Ed., Harcourt College Press, 1988.

[7] G. H. Golub, C. F. Van Loan, *Matrix Computations*, 3rd ed, Johns Hopkins Univ. Press, 1996.

[8] O. Axelsson, *Iterative Solution Methods*, Cambridge Univ. Press, 1996.

[9] W. H. Press, S. A. Teukolsky, W. T. Vetterling, B. P. Flannery, *Numerical Recipes in C*, 2nd Ed., Cambridge Univ. Press, 1993.