A morphism of Green functors for S_3

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Abstract

The purpose of this note is to compute the morphism from the crossed Burnside ring Green functor to the Grothendieck ring Green functor for the group algebra of the symmetric group of three letters over the filed of the rational numbers.

Key words : Crossed G-sets, Drinfel'd double, symmetric group.

1 Crossed Burnside rings and Drinfel'd doubles

(1.1) Crossed Burnside ring Green functors. We recall the crossed Burnside ring Green functor $X\Omega(*, G^c)$ in terms of subgroups of G (see [OY04]). If H is a subgroup of G, then let $X\Omega(H, G^c)$ be the $\mathbb{Z}$-module defined by

$$X\Omega(H,G^c) = \langle (H/D)_s \mid D \in [H \setminus S(H)],\ s \in [H \setminus C_G(D)] \rangle_{\mathbb{Z}},$$

where $S(H)$ is the family of all subgroups of H. Then the assignment

$$H(\leq G) \longmapsto X\Omega(H, G^c)$$

gives a Mackey functor equipped with

$$\begin{array}{llllll} ind_K^H & : & X\Omega(K,G^c) \longrightarrow X\Omega(H,G^c) & : & (K/D)_s \longmapsto (H/D)_s, \\ res_K^H & : & X\Omega(H,G^c) \longrightarrow X\Omega(K,G^c) & : & (H/D)_s \longmapsto \displaystyle\sum_{g\in[K\setminus H/D]} (K/K \cap {}^gD)_{^gs}, \\ con_{H,g} & : & X\Omega(H,G^c) \longrightarrow X\Omega({}^gH,G^c) & : & (H/D)_s \longmapsto ({}^gH/{}^gD)_{^gs}, \end{array}$$

where $D \leq K \leq H \leq G$ and $g \in G$. We define a product

$$((H/D)_s, (H/E)_t) \mapsto (H/D)_s \cdot (H/E)_t$$

on $\mathbb{Z}$-module $X\Omega(H, G^c)$ by

$$(H/D)_s \cdot (H/E)_t = \sum_{x\in[D\setminus H/E]} (H/D \cap {}^xE)_{s \cdot {}^xt}.$$

Then for any subgroup H of G, $\mathbb{Z}$-module $X\Omega(H, G^c)$ has a structure of associative ring with unit $(H/H)_1$, which is compatible with the Mackey structure, in the following sense:

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- If $x \in G$, and K is a subgroup of G, then the conjugation by x is a morphism of ring with unit from $X\Omega(H, G^c)$ to $X\Omega({}^xH, G^c)$.
- If $K \leq H$ are subgroups of G, then res^H_K is a morphism of ring with unit from $X\Omega(H, G^c)$ to $X\Omega(K, G^c)$.
- In the same conditions, if $a \in X\Omega(H, G^c)$ and $b \in X\Omega(K, G^c)$, then

$$\begin{aligned} a \cdot ind^H_K(b) &= ind^H_K(res^H_K(a) \cdot b) \\ ind^H_K(b) \cdot a &= ind^H_K(b \cdot res^H_K(a)). \end{aligned}$$

Hence $X\Omega(*, G^c)$ is a Green functor for G over $\mathbb{Z}$.

(1.2) Drinfel'd doubles. The Drinfel'd double $D(G)$ is the Hopf algebra given by the smash product of the group algebra $\mathbb{C}G$, with its Hopf algebraic dual $(\mathbb{C}G)^*$. In particular, for each subgroup H of G, there is a subalgebra

$$D_G(H) = \sum_{g \in G,\, h \in H} \mathbb{C}\phi_g h.$$

It is useful to notice that for any two subgroups $L \leq H$ of G we have an isomorphism of right $D_G(L)$-modules

$$D_G(H) = \sum_{h \in H/L} h \otimes D_G(L),$$

with $h \otimes D_G(L) \cong D_G(L)$ as a right $D_G(L)$-module for each left coset representative h of H/L. That is , $D_G(H)$ is a free right $D_G(L)$-module.

(1.3) Vector bundles. Fix a finite left G-set X. A *G-equivariant $\mathbb{C}$-vector bundle U* on X is a collection of finite dimensional vector spaces $\{U_x\}_{x \in X}$, together with a representation of G on their direct sum $\sum_{x \in X} Ux$ such that $g \cdot U_x = U_{gx}$. A *x-component* or *fiber* of U is the vector space U_x. If u is an element of the $\mathbb{C}G$-module $\sum_{x \in X} U_x$, we write $u = \sum_{x \in X} u_x$, where $u_x \in U_x$ for each $x \in X$. A *morphism* $f : U \to V$ of G-vector bundles U and V is a $\mathbb{C}G$-module homomorphism $f : \sum_{x \in X} U_x \to \sum_{x \in X} V_x$ which preserves fibers. The indecomposable (respectively, irreducible) G-vector bundles on X are indexed by pairs (V, x) where x is a representative of an orbit of X and V is an indecomposable (respectively, irreducible) kG_x-module. The product $U \otimes V$ of H-vector bundles U and V is defined to be

$$(U \otimes V)_g = \sum_{x \in G} U_x \otimes V_{x^{-1}g}$$

for $g \in G$. The sum of these components has $\mathbb{C}H$-module structure given by the action of H on the tensor product of the underlying $\mathbb{C}H$-modules of U and V. It is easy to check that $h \cdot (U \otimes V)_g = (U \otimes V)_{{}^hg}$ for $h \in H$.

(1.4) Witherspoon's Green functors. Witherspoon defined the Green functor of the Grothendieck ring of Drinfel'd (quantum) double $D(G)$ of the group algebra kG over a field k [Wi96].

We denote three kinds of morphism as follows:

$$\begin{array}{llllll} Dres^H_K & : R(D_G(H)) & \longrightarrow & R(D_G(K)) & ; \ U & \longmapsto U\downarrow_{D_G(K)}, \\ Dind^H_K & : R(D_G(K)) & \longrightarrow & R(D_G(H)) & ; \ V & \longmapsto D_G(H) \otimes_{D_G(K)} V, \\ Dconj_{H,g} & : R(D_G(H)) & \longrightarrow & R(D_G({}^gH)) & ; \ U & \longmapsto {}^gU = gD_G(H) \otimes_{D_G(H)} U, \end{array}$$

where $U\downarrow_{D_G(K)}$ is a $D_G(K)$-module by restriction of the action from $D_G(H)$ to $D_G(K)$. The algebras $R(D_G(H))$, for all subgroups H of G, constitute a Green functor for G.

(1.5) Morphisms of Green functors. A *morphism* θ from a Mackey functor M to a Mackey functor N consists of a collection of morphisms of $\mathcal{O}$-modules $\theta_H : M(H) \to N(H)$, for $H \leq G$, such that if $K \leq H$ and $x \in G$, the squares

$$\begin{array}{ccc} M(K) & \xrightarrow{\theta_K} & N(K) \\ {\scriptstyle t_K^H}\downarrow & & \downarrow{\scriptstyle t_K^H} \\ M(H) & \xrightarrow[\theta_H]{} & N(H), \end{array} \qquad \begin{array}{ccc} M(K) & \xrightarrow{\theta_K} & N(K) \\ {\scriptstyle r_K^H}\uparrow & & \uparrow{\scriptstyle r_K^H} \\ M(H) & \xrightarrow[\theta_H]{} & N(H), \end{array} \qquad \begin{array}{ccc} M(H) & \xrightarrow{\theta_H} & N(H) \\ {\scriptstyle c_{x,H}}\downarrow & & \downarrow{\scriptstyle c_{x,H}} \\ M({}^xH) & \xrightarrow[\theta_{{}^xH}]{} & N({}^xH) \end{array}$$

are commutative. A *morphism* ϕ from the Green functor A to the Green functor B is a morphism of Mackey functors such that, for any subgroup H of G, the morphism ϕ_H is a morphism of rings. The morphism ϕ is said to be unitary if the morphism ϕ_H preserves unit for all H. There is a natural homomorphism θ from Burnside Green functor Ω to Grothendieck ring Green functor R :

$$\theta_X : \Omega(X) \to R(X) : (\varphi : Y \to X) \mapsto \{\mathbb{C}[\varphi^{-1}(x)]\}_{x\in X}.$$

(1.6) Theorem. ([Bo03]) *Let A be a Green functor for G over a commutative ring $\mathcal{O}$ and Γ a crossed G-monoid. Then the functor A_Γ is a Green functor for G over $\mathcal{O}$, with unit ε_{A_Γ}. Moreover the correspondence $A \mapsto A_\Gamma$ is an endo-functor of the category of Green functor for G over $\mathcal{O}$.*

Theorem 1.6 leads to the following lemma.

(1.7) Lemma. *Let H be a subgroup of G. Then a homomorphism*

$$\theta_{G/H\times G^c} : X\Omega(H, G^c) \to R_{\mathbb{C}}(D_{G^c}(H))$$

induced by the natural morphism $\theta : \Omega \to R$ of Green functors is a unitary ring homomorphism. The homomorphism $\theta_{G/H\times G^c}$ assigns to an element $(H/D)_g$ of basis of $X\Omega(H, G^c)$ H-vector bundle

$$[H/D]_g = \sum_{xC_H(g)\in G/C_H(g)} \mathbb{C}[m_s^{-1}(xC_H(g))],$$

where m_s is a canonical G-map from G/D to $G/C_H(s)$.

(1.8) Example. Put $G = S_3 = \langle s, t \,|\, s^3 = t^2 = e, {}^ts = s^2\rangle$ and $H = e :=< e >$. Let $\theta : \Omega \longrightarrow R$ be a natural morphism defined by $\theta_X : \Omega(X) \longrightarrow R(X)$. Then we have $\theta_{X\times G^c} : \Omega(X\times G^c) \longrightarrow R(X\times G^c)$. For transitive G-set G/H, we have $\theta_{G/H} : \Omega(G/H) \longrightarrow R(G/H)$. By Lemma 1.7, we have $\theta_{G/H\times G^c} : \Omega(G/H \times G^c) \longrightarrow R(G/H \times G^c)$. Since $G/H \times G^c \cong \coprod_{g\in[H\backslash G^c]} G/H_g$ as G-sets, we have $\Omega(G/H \times G^c) \cong$

$$\Omega\left(\coprod_{g\in[H\backslash G^c]} G/H_g\right) \cong \bigoplus_{g\in[H\backslash G^c]} \Omega(G/H_g) \cong \bigoplus_{g\in[H\backslash G^c]} \left(\bigoplus_{xH_g\in G/H_g} \Omega(G_{xH_g})\right)^G.$$

So the natural homomorphism $\theta_{G/H\times G^c} : \Omega(G/H\times G^c) \longrightarrow R(G/H\times G^c)$ of Green functors is written by the following formula:

$$\bigoplus_{g\in[H\backslash G^c]}\left(\bigoplus_{xH_g\in G/H_g}\Omega\left(G_{xH_g}\right)\right)^G \xrightarrow{\oplus\theta_{g,xH_g}} \bigoplus_{g\in[H\backslash G^c]}\left(\bigoplus_{xH_g\in G/H_g}R\left(G_{xH_g}\right)\right)^G .$$

We have that $H\setminus G^c =< e > \backslash G = \{e\}\coprod\{t\}\coprod\{st\}\coprod\{s^2t\}\coprod\{s\}\coprod\{s^2\}$ and $[H\setminus G^c] = [< e > \backslash G] = \{e\}\coprod\{t\}\coprod\{st\}\coprod\{s^2t\}\coprod\{s\}\coprod\{s^2\}$.
So $\Omega(G/H\times G^c) \cong \bigoplus_{g\in G}\left(\bigoplus_{xH_g\in G/H_g}\Omega\left(G_{xH_g}\right)\right)^G \cong \bigoplus_{g\in G}\left(\bigoplus_{x\in G/e}\Omega\left(G_x\right)\right)^G$ and

$R(G/H\times G^c) \cong \bigoplus_{g\in G}\left(\bigoplus_{xH_g\in G/H_g}R\left(G_{xH_g}\right)\right)^G \cong \bigoplus_{g\in G}\left(\bigoplus_{x\in G/e}R\left(G_x\right)\right)^G$. Hence the homomorphism $\theta_{G/e\times G^c}$ is a sum of the natural homomorphism between these modules.

(1.9) Example. Put $G = S_3$ and $H = C :=< t >$. Then $H\setminus G^c = C\setminus G = \{e\}\coprod\{t\}\coprod\{st, s^2t\}\coprod\{s, s^2\}$ and $[H\setminus G^c] = [C\setminus G] = \{e\}\coprod\{t\}\coprod\{st\sim s^2t\}\coprod\{s\sim s^2\}$.
So $\Omega(G/H\times G^c)\cong$

$$\left(\bigoplus_{xH_e\in G/H_e}\Omega\left(G_{xH_e}\right)\right)^G \oplus \left(\bigoplus_{xH_t\in G/H_t}\Omega\left(G_{xH_t}\right)\right)^G \oplus \left(\bigoplus_{xH_{st}\in G/H_{st}}\Omega\left(G_{xH_{st}}\right)\right)^G \oplus \left(\bigoplus_{xH_{s^2t}\in G/H_{s^2t}}\Omega(G_{xH_{s^2t}})\right)^G .$$

$$\left(\bigoplus_{xH_e\in G/H_e}\Omega\left(G_{xH_e}\right)\right)^G \cong \left(\bigoplus_{xC_e\in G/C_e}\Omega\left(G_{xC_e}\right)\right)^G \cong \left(\bigoplus_{xC\in G/C}\Omega\left(G_{xC}\right)\right)^G$$

$\cong \left(\Omega\left(G_{eC}\right)\oplus\Omega\left(G_{sC}\right)\oplus\Omega\left(G_{s^2C}\right)\right)^G$
$\cong \left(\Omega\left(G_{eC}\right)\oplus\Omega\left(G_{sC}\right)\oplus\Omega\left(G_{s^2C}\right)\right)^G$
$\cong \left(\Omega\left(C\right)\oplus\Omega\left(C\right)\oplus\Omega\left(C\right)\right)^G$
$\cong \Omega\left(C\right).$

$$\left(\bigoplus_{xH_t\in G/H_t}\Omega\left(G_{xH_t}\right)\right)^G \cong \left(\bigoplus_{xC_t\in G/C_t}\Omega\left(G_{xC_t}\right)\right)^G \cong \left(\bigoplus_{xC\in G/C}\Omega\left(G_{xC}\right)\right)^G$$

$\cong \left(\Omega\left(G_{eC}\right)\oplus\Omega\left(G_{sC}\right)\oplus\Omega\left(G_{s^2C}\right)\right)^G$
$\cong \left(\Omega\left(G_{eC}\right)\oplus\Omega\left(G_{sC}\right)\oplus\Omega\left(G_{s^2C}\right)\right)^G$
$\cong \left(\Omega\left(C\right)\oplus\Omega\left(C\right)\oplus\Omega\left(C\right)\right)^G$
$\cong \Omega\left(C\right).$

$$\left(\bigoplus_{xH_{st}\in G/H_{st}}\Omega\left(G_{xH_{st}}\right)\right)^G \cong \left(\bigoplus_{xC_{st}\in G/C_{st}}\Omega\left(G_{xC_{st}}\right)\right)^G \cong \left(\bigoplus_{x\in G/e}\Omega\left(G_x\right)\right)^G$$

$\cong \left(\Omega\left(G_e\right)\oplus\Omega\left(G_t\right)\oplus\Omega\left(G_{st}\right)\oplus\Omega\left(G_{s^2t}\right)\oplus\Omega\left(G_s\right)\oplus\Omega\left(G_{s^2}\right)\right)^G$
$\cong \left(\Omega\left(< e >\right)\oplus\Omega\left(< e >\right)\oplus\Omega\left(< e >\right)\oplus\Omega\left(< e >\right)\oplus\Omega\left(< e >\right)\oplus\Omega\left(< e >\right)\right)^G$
$\cong \left(\Omega\left(e\right)\oplus\Omega\left(e\right)\oplus\Omega\left(e\right)\oplus\Omega\left(e\right)\oplus\Omega\left(e\right)\oplus\Omega\left(e\right)\right)^G$
$\cong \Omega\left(e\right).$

$$\left(\bigoplus_{xH_{s^2t}\in G/H_{s^2t}}\Omega\left(G_{xH_{s^2t}}\right)\right)^G\cong\left(\bigoplus_{xC_{s^2t}\in G/C_{s^2t}}\Omega\left(G_{xC_{s^2t}}\right)\right)^G\cong\left(\bigoplus_{x\in G/e}\Omega\left(G_x\right)\right)^G$$
$$\cong\left(\Omega\left(G_e\right)\oplus\Omega\left(G_t\right)\oplus\Omega\left(G_{st}\right)\oplus\Omega\left(G_{s^2t}\right)\oplus\Omega\left(G_s\right)\oplus\Omega\left(G_{s^2}\right)\right)^G$$
$$\cong\left(\Omega\left(<e>\right)\oplus\Omega\left(<e>\right)\oplus\Omega\left(<e>\right)\oplus\Omega\left(<e>\right)\oplus\Omega\left(<e>\right)\oplus\Omega\left(<e>\right)\right)^G$$
$$\cong\left(\Omega\left(e\right)\oplus\Omega\left(e\right)\oplus\Omega\left(e\right)\oplus\Omega\left(e\right)\oplus\Omega\left(e\right)\oplus\Omega\left(e\right)\right)^G$$
$$\cong\Omega\left(e\right).$$

Hence the homomorphism $\theta_{G/C\times G^c}$ is induced by the direct sum.

(1.10) Example. Put $G=S_3$ and $H=D:=<s>$. Then $H\setminus G^c=D\setminus G=\{e\}\coprod\{s\}\coprod\{s^2\}\coprod\{t,st,s^2t\}$ and $[H\setminus G^c]=[D\setminus G]=\{e\}\coprod\{s\}\coprod\{s^2\}\coprod\{t\sim s^2t\sim st\}$, where ${}^st=s^2t$, ${}^ss^2t=s$, ${}^sst=t$.
So $\Omega(G/H\times G^c)\cong$

$$\left(\bigoplus_{xH_e\in G/H_e}\Omega\left(G_{xH_e}\right)\right)^G\oplus\left(\bigoplus_{xH_s\in G/H_s}\Omega\left(G_{xH_s}\right)\right)^G\oplus\left(\bigoplus_{xH_{s^2}\in G/H_{s^2}}\Omega(G_{xH_{s^2}})\right)^G\oplus\left(\bigoplus_{xH_t\in G/H_t}\Omega\left(G_{xH_t}\right)\right)^G.$$

$$\left(\bigoplus_{xH_e\in G/H_e}\Omega\left(G_{xH_e}\right)\right)^G\cong\left(\bigoplus_{xD_e\in G/D_e}\Omega\left(G_{xD_e}\right)\right)^G\cong\left(\bigoplus_{xD\in G/D}\Omega\left(G_{xD}\right)\right)^G$$
$$\cong\left(\Omega\left(G_{eD}\right)\oplus\Omega\left(G_{tD}\right)\right)^G$$
$$\cong\left(\Omega\left(G_{eD}\right)\oplus\Omega\left(G_{tD}\right)\right)^G$$
$$\cong\left(\Omega\left(D\right)\oplus\Omega\left(D\right)\right)^G$$
$$\cong\Omega\left(D\right).$$

$$\left(\bigoplus_{xH_s\in G/H_s}\Omega\left(G_{xH_s}\right)\right)^G\cong\left(\bigoplus_{xD_s\in G/D_s}\Omega\left(G_{xD_s}\right)\right)^G\cong\left(\bigoplus_{xD\in G/D}\Omega\left(G_{xD}\right)\right)^G$$
$$\cong\left(\Omega\left(G_{eD}\right)\oplus\Omega\left(G_{tD}\right)\right)^G$$
$$\cong\left(\Omega\left(G_{eD}\right)\oplus\Omega\left(G_{tD}\right)\right)^G$$
$$\cong\left(\Omega\left(D\right)\oplus\Omega\left(D\right)\right)^G$$
$$\cong\Omega\left(D\right).$$

$$\left(\bigoplus_{xH_e\in G/H_{s^2}}\Omega\left(G_{xH_{s^2}}\right)\right)^G\cong\left(\bigoplus_{xD_{s^2}\in G/D_{s^2}}\Omega\left(G_{xD_{s^2}}\right)\right)^G\cong\left(\bigoplus_{xD\in G/D}\Omega\left(G_{xD}\right)\right)^G$$
$$\cong\left(\Omega\left(G_{eD}\right)\oplus\Omega\left(G_{tD}\right)\right)^G$$
$$\cong\left(\Omega\left(G_{eD}\right)\oplus\Omega\left(G_{tD}\right)\right)^G$$
$$\cong\left(\Omega\left(D\right)\oplus\Omega\left(D\right)\right)^G$$
$$\cong\Omega\left(D\right).$$

$$\left(\bigoplus_{xH_t\in G/H_t}\Omega\left(G_{xH_t}\right)\right)^G\cong\left(\bigoplus_{xD_t\in G/D_t}\Omega\left(G_{xD_t}\right)\right)^G\cong\left(\bigoplus_{x\in G/e}\Omega\left(G_x\right)\right)^G$$
$$\cong\left(\Omega\left(G_e\right)\oplus\Omega\left(G_t\right)\oplus\Omega\left(G_{st}\right)\oplus\Omega\left(G_{s^2t}\right)\oplus\Omega\left(G_s\right)\oplus\Omega\left(G_{s^2}\right)\right)^G$$
$$\cong\left(\Omega\left(e\right)\oplus\Omega\left(e\right)\oplus\Omega\left(e\right)\oplus\Omega\left(e\right)\oplus\Omega\left(e\right)\oplus\Omega\left(e\right)\right)^G$$
$$\cong\Omega\left(e\right).$$

Hence the homomorphism $\theta_{G/C\times G^c}$ is induced by the direct sum.

(1.11) Example. Put $G = S_3$ and $H = G$. Then $H\backslash G^c = G\backslash G = \{e\} \coprod \{t, st, s^2t\} \coprod \{s, s^2\}$ and $[H \setminus G^c] = [G \setminus G] = \{e\} \coprod \{t \sim s^2t \sim st\} \coprod \{s \sim s^2\}$, where ${}^st = s^2t$, ${}^ss^2t = s$, ${}^sst = t, {}^ts = s^2$.
So $\Omega(G/H \times G^c) \cong$

$$\left(\bigoplus_{xH_e \in G/H_e} \Omega\left(G_{xH_e}\right)\right)^G \oplus \left(\bigoplus_{xH_t \in G/H_t} \Omega\left(G_{xH_t}\right)\right)^G \oplus \left(\bigoplus_{xH_s \in G/H_s} \Omega\left(G_{xH_s}\right)\right)^G.$$

$$\left(\bigoplus_{xH_e \in G/H_e} \Omega\left(G_{xH_e}\right)\right)^G \cong \left(\bigoplus_{xG_e \in G/G_e} \Omega\left(G_{xG_e}\right)\right)^G \cong \left(\bigoplus_{xG \in G/G} \Omega\left(G_{xG}\right)\right)^G$$
$$\cong \left(\Omega\left(G_{xG}\right)\right)^G$$
$$\cong \Omega\left(G\right).$$

$$\left(\bigoplus_{xH_t \in G/H_t} \Omega\left(G_{xH_t}\right)\right)^G \cong \left(\bigoplus_{xG_t \in G/G_t} \Omega\left(G_{xG_t}\right)\right)^G \cong \left(\bigoplus_{xC \in G/C} \Omega\left(G_{xC}\right)\right)^G$$
$$\cong \left(\Omega\left(G_{eC}\right) \oplus \Omega\left(G_{sC}\right) \oplus \Omega\left(G_{s^2C}\right)\right)^G$$
$$\cong \left(\Omega\left(< t >\right) \oplus \Omega\left(< s^2t >\right) \oplus \Omega\left(< st >\right)\right)^G$$
$$\cong \Omega\left(C\right).$$

$$\left(\bigoplus_{xH_s \in G/H_s} \Omega\left(G_{xH_s}\right)\right)^G \cong \left(\bigoplus_{xG_s \in G/G_s} \Omega\left(G_{xG_s}\right)\right)^G \cong \left(\bigoplus_{xD \in G/D} \Omega\left(G_{xD}\right)\right)^G$$
$$\cong \left(\Omega\left(G_{eD}\right) \oplus \Omega\left(G_{tD}\right)\right)^G$$
$$\cong \left(\Omega\left(< s >\right) \oplus \Omega\left(< s^2 >\right)\right)^G$$
$$\cong \Omega\left(D\right).$$

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